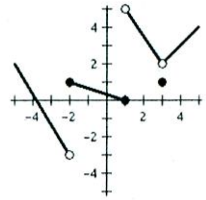
The graph of f is shown on the right. Evaluate the following limits. Write "DNE" if a limit does not exist. **SCORE: _____ / 3 PTS**



[a] $\displaystyle\lim_{x\to 3}\frac{x^2}{2+f(x)}$

$$= \boxed{\frac{\displaystyle\lim_{x\to 3} x^2}{\displaystyle\lim_{x\to 3} 2 + \lim_{x\to 3} f(x)}} \quad 1$$

$$= \frac{9}{2+2} = \boxed{\frac{9}{4}} \quad 1$$

[b] $\displaystyle\lim_{x\to -2^-} f(x)$

$$= \boxed{-3} \quad 1$$

Prove that $\displaystyle\lim_{x\to 0} x^4\cos\frac{1}{x^2} = 0$. **SCORE: _____ / 3 PTS**

$$\boxed{-x^4 \le x^4\cos\frac{1}{x^2} \le x^4} \quad 1$$

$$\boxed{\lim_{x\to 0} -x^4 = 0} \quad \tfrac{1}{2}$$

$$\boxed{\lim_{x\to 0} x^4 = 0} \quad \tfrac{1}{2}$$

OK IF TOGETHER IN A COMPOUND EQUALITY

SO $\boxed{\displaystyle\lim_{x\to 0} x^4\cos\frac{1}{x^2} = 0 \text{ BY SQUEEZE THEOREM}}$ 1

If $\displaystyle\lim_{r\to -1}\frac{7-ar-r^5}{1+r}$ exists, find the value of a. **SCORE: _____ / 2 PTS**

SINCE $\displaystyle\lim_{r\to -1}(1+r)=0$, THE ORIGINAL LIMIT EXISTS ONLY

IF $\displaystyle\lim_{r\to -1}(7-ar-a^5)=0$ IE. $\boxed{7+a+1=0}$ 1

OK IF SIMPLIFIED

$\boxed{a=-8}$ 1

Using complete sentences and proper mathematical notation, write the formal definition of "vertical asymptote". **SCORE: _____ / 2 PTS**

f HAS A VERTICAL ASYMPTOTE AT a IFF
$\lim_{x \to a^+} f(x) = \infty$ OR $\lim_{x \to a^+} f(x) = -\infty$
OR $\lim_{x \to a^-} f(x) = \infty$ OR $\lim_{x \to a^-} f(x) = -\infty$

GRADED BY ME

Evaluate the following limits. Write "DNE" if a limit does not exist. **SCORE: _____ / 7 PTS**

[a] $\lim_{y \to -3} \frac{y^2 - 2y - 15}{2y^2 + 3y - 9}$ $\frac{0}{0}$

$= \lim_{y \to -3} \frac{(y+3)(y-5)}{(y+3)(2y-3)}$

$= \boxed{\lim_{y \to -3} \frac{y-5}{2y-3}}$ 1

$= \frac{-8}{-9} = \boxed{\frac{8}{9}}$ $\frac{1}{2}$

[b] $\lim_{b \to 4} \frac{b - \sqrt{b+12}}{8 - 2b}$ $\frac{0}{0}$

$= \boxed{\lim_{b \to 4} \frac{b^2 - b - 12}{(8-2b)(b + \sqrt{b+12})}}$ 1

$= \lim_{b \to 4} \frac{(b-4)(b+3)}{-2(b-4)(b + \sqrt{b+12})}$

$= \boxed{\lim_{b \to 4} \frac{b+3}{-2(b + \sqrt{b+12})}}$ 1

$= \frac{7}{(-2)8} = \boxed{-\frac{7}{16}}$ $\frac{1}{2}$

[c] $\lim_{t \to -2} \frac{\frac{6}{1-t} - \frac{5}{t+3}}{t^2 + 4}$

$= \frac{\frac{6}{3} - \frac{5}{1}}{4+4}$

$= \frac{2-5}{8}$

$= \boxed{-\frac{3}{8}}$ 1

[d] $\lim_{x \to 5} f(x)$ where $f(x) = \begin{cases} 2x+1, & \text{if } x < -3 \\ 1-x, & \text{if } -3 < x < 5 \\ x-9, & \text{if } x > 5 \end{cases}$

$\lim_{x \to 5^-} f(x) = \boxed{\lim_{x \to 5^-} (1-x) = -4}$ $\frac{1}{2}$ ← REQUIRED

$\lim_{x \to 5^+} f(x) = \boxed{\lim_{x \to 5^+} (x-9) = -4}$ ← REQUIRED

$\frac{1}{2}$

SO $\boxed{\lim_{x \to 5} f(x) = -4}$ 1